

# Two-Photon Airy Disk

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## Abstract

We report an experimental observation of quantum Airy disk diffraction pattern using an entangled two-photon source. In contrast to the previous quantum lithography experiments where the subwavelength diffraction patterns were observed in the far field limit, we perform the Fraunhofer diffraction experiment with a convex lens. The experimental result shows that the two-photon Airy disk is provided with the super-resolution spot, which surpasses the classical diffraction limit. In particular, the spot size can be well controlled by the focal length, which adapted to optical super-focusing. Our experiment can promote potential application of quantum lithography.

Focusing effect of a light beam has potential applications in many areas such as micro-optical fabrication, high-precision processing, high-resolution imaging and photolithography, etc. In classical optics, the minimum size of the focused light spot is governed by the diffraction limit. Recently, theoretical and experimental studies showed that quantum interference with a multiphoton entangled state can surpass the classical diffraction limit[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Theoretically, Jacobson et al. [1] proved that the de Broglie wavelength of  $N$ -photon packet is  $\lambda/N$ , where  $\lambda$  and  $N$  are the wavelength and number of the constituent photons, respectively. Boto et al. [2] proposed a scenario of quantum interferometric optical lithography with  $N$ -photon entangled state to beat the diffraction limit. The theoretical proposals were then implemented in the experiments using two-photon entangled state generated by spontaneous parametric down-conversion(SPDC)[3, 4, 7, 9]. In a frequency degenerate type-II SPDC, a pump photon with frequency  $\omega_p$  impinges on a  $\beta$ -BaB<sub>2</sub>O<sub>4</sub> (BBO) crystal to generate a pair of entangled photons with the frequencies  $\omega_s \approx \omega_i = \omega_p/2$  in different polarized modes. When the photon pairs are projected onto a double slit, the interference-diffraction pattern can be observed by the two-photon coincidence measurement. The pattern exhibits a smaller spatial modulation period which beats the classical diffraction limit by a factor of two[4]. Moreover, it was noticed recently that the two-photon subwavelength interference for a double-slit can also be realized in a similar way with a well-designed interferometer driven by a thermal light source[11].

In the quantum lithography experiments with a two-photon entangled state, the resolution improvement of the interference-diffraction pattern exists in the far-field limit where Fraunhofer diffraction occurs[3, 4, 7, 9]. However, the pattern to be observed in the far-field has been expanded in a large scale, which is generally larger than the object size. Hence the far-field pattern would not be appropriate for the technical application of photolithography. On the other hand, our recent work[12] showed that there is no net improvement of the transverse resolution in the near-field Fresnel diffraction, such as two-photon Talbot self-imaging. Instead, the two-photon subwavelength effect is reflected in twice the propagation distance for one-photon diffraction.

In this paper we consider two-photon Fraunhofer diffraction using a lens. We find that in this scheme the resolution of the diffraction pattern is enhanced to beat the classical diffraction limit, as the same quantum lithography for the far field case. At the same time, the spot size of the pattern can be well controlled by the focal length.

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Let a two-photon entangled state be

$$|\Psi\rangle = \int d\mathbf{r}_1 d\mathbf{r}_2 C(\mathbf{r}_1, \mathbf{r}_2) a_1^\dagger(\mathbf{r}_1) a_2^\dagger(\mathbf{r}_2) |0\rangle, \quad (1)$$

where  $a_j^\dagger (j = 1, 2)$  are the photon creation operators for the two SPDC modes, and  $\mathbf{r}_j (j = 1, 2)$  are their transverse coordinates across the beam.  $C(\mathbf{r}_1, \mathbf{r}_2) \sim \langle 0 | E_{s1}^{(+)}(\mathbf{r}_1) E_{s2}^{(+)}(\mathbf{r}_2) | \Psi \rangle$  characterizes the two-photon wavepacket with the field operators  $E_j^{(+)} (j = s1, s2)$  of mode  $j$  in the source plane. After propagation, the diffraction field in the observation plane is given by

$$E^{(+)}(\mathbf{r}) = \int d\mathbf{r}_0 h(\mathbf{r}, \mathbf{r}_0) E_s^{(+)}(\mathbf{r}_0), \quad (2)$$

where  $h(\mathbf{r}, \mathbf{r}_0)$  is the impulse response function. So the two-photon wavepacket in the observation plane is obtained as

$$\begin{aligned} & \langle 0 | E_1^{(+)}(\mathbf{r}_1) E_2^{(+)}(\mathbf{r}_2) | \Psi \rangle \\ &= \int d\mathbf{r}'_0 d\mathbf{r}''_0 h_1(\mathbf{r}_1, \mathbf{r}'_0) h_2(\mathbf{r}_2, \mathbf{r}''_0) C(\mathbf{r}'_0, \mathbf{r}''_0), \end{aligned} \quad (3)$$

where  $h_j$  is the impulse response function for the field mode  $j$ . The two-photon coincidence counting rate reads

$$\begin{aligned} R(\mathbf{r}_1, \mathbf{r}_2) &\propto \langle \Psi | E_1^{(-)}(\mathbf{r}_1) E_2^{(-)}(\mathbf{r}_2) E_2^{(+)}(\mathbf{r}_2) E_1^{(+)}(\mathbf{r}_1) | \Psi \rangle \\ &= |\langle 0 | E_1^{(+)}(\mathbf{r}_1) E_2^{(+)}(\mathbf{r}_2) | \Psi \rangle|^2. \end{aligned} \quad (4)$$

For simplicity, we consider an ideal two-photon entangled state at the source, which satisfies  $C(\mathbf{r}'_0, \mathbf{r}''_0) = \delta(\mathbf{r}'_0 - \mathbf{r}''_0)$ . Eq.(3) is written as

$$\langle 0 | E_1^{(+)}(\mathbf{r}_1) E_2^{(+)}(\mathbf{r}_2) | \Psi \rangle = \int d\mathbf{r}_0 h_1(\mathbf{r}_1, \mathbf{r}_0) h_2(\mathbf{r}_2, \mathbf{r}_0). \quad (5)$$

The impulse response function from the front to the back focal planes of a thin lens is given by  $h_f(\mathbf{r}, \mathbf{r}_0) = 1/(i\lambda f) \times \exp[i4\pi f/\lambda] \exp[-i2\pi(\mathbf{r} \cdot \mathbf{r}_0)/(\lambda f)]$ , where  $\lambda$  is the wavelength and  $f$  is the focal length of the lens. We put the two-photon source at the front focal plane of the lens and the detector at the back one. Assuming a mask object of the transmittance function  $P(\mathbf{r}_0)$  (note that  $P^2(\mathbf{r}_0) = P(\mathbf{r}_0)$ ) is placed at the source, we obtain the two-photon coincidence counting rate

$$\begin{aligned} R(\mathbf{r}_1, \mathbf{r}_2) &\propto \left| \int P(\mathbf{r}_0) \exp[-i2\pi(\mathbf{r}_1 + \mathbf{r}_2) \cdot \mathbf{r}_0/(\lambda f)] d\mathbf{r}_0 \right|^2 \\ &\propto \left| \tilde{P}[2\pi(\mathbf{r}_1 + \mathbf{r}_2)/(\lambda f)] \right|^2, \end{aligned} \quad (6)$$

where  $\tilde{P}$  is the Fourier transform of function  $P$ .

Let a circle aperture of radius  $a$  be placed at the front focal plane of the lens, we then calculate the normalized two-photon coincidence counting distribution for  $\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}$  in the back focal plane

$$R(\mathbf{r})/R(0) = \left| \frac{2J_1(\frac{2\pi}{\lambda f} 2ar)}{\frac{2\pi}{\lambda f} 2ar} \right|^2, \quad (7)$$

where  $R(\mathbf{r}) \equiv R(\mathbf{r}, \mathbf{r})$ ;  $J_1$  is the Bessel function of the first kind and  $r = |\mathbf{r}|$ . This is the well-known Airy disk, a focused spot pattern when a plane wave incident on a circle lens or a diffraction pattern of a circular source in the far-field limit. But the present Airy disk has a two times resolution of the classical one, and hence the size of the central spot is reduced to half.

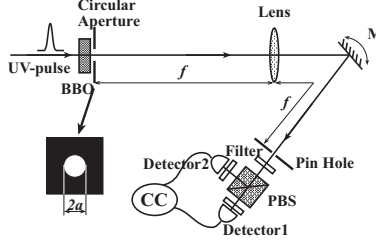


Figure 1: Schematic diagram of experimental setup. Photon pairs are generated by a BBO crystal pumped by a UV pulse laser. A circular aperture closed to the BBO and a pin hole are placed in the front and back focal planes of a lens, respectively. PBS is a polarizing beam splitter and M is a mirror. Coincidence counting is implemented by two detectors.

To demonstrate the two-photon Airy disk described above, we perform an experiment which is sketched in Fig. 1. The UV-pulse beam is provided by a second-harmonic of a Ti:sapphire femtosecond laser (Mira-900 Coherent Inc.) with the central wavelength  $\lambda = 400\text{nm}$  and repetition rate 76 MHz. The UV-pulse is used to pump a  $5 \times 5 \times 2\text{mm}$  type-II BBO crystal to generate collinear orthogonally polarized photons via the spontaneous parametric down-conversion (SPDC) process. A circular aperture is placed immediately after the crystal and at the front focal plane of the lens. A pin hole is placed at the back focal plane. After passing through the pin hole, the pump beam is blocked by a cutoff filter. However, the entangled photon pairs are separated by a polarizing beam splitter (PBS) and then detected by two single-photon detectors (Perkin-Elmer SPCM-AQR-14). Both detectors are preceded by 10nm bandwidth interference filters centered at the degenerate wavelength of 800nm. The pin hole, PBS and the coincidence circuit work together as a two-photon detector. For the sake of convenience, instead of moving the two detectors together, we rotate the mirror M to “scan” the diffraction pattern across the detector surface[4]. The time window for the coincidence counts is 2ns.

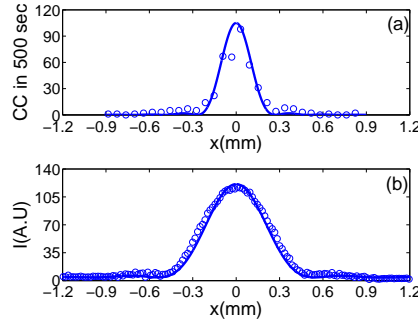


Figure 2: Experimental observation of the Airy disk patterns with (a) a two-photon entangled source, and (b) a classical coherent source, where the experimental curves in (a) and (b) are recorded by the two-photon coincidence measurement and the intensity measurement, respectively. The open circles are the experimental data and the solid lines are the theoretical curves.

In our scheme, the diameter of the circular aperture is  $2a = 0.9\text{mm}$ , and the focal length of the lens is  $f = 50\text{cm}$ . The experimental results are shown in Fig. 2, where the experimental data and theoretical curves are indicated by open circles and solid lines, respectively. Figure 2(a) shows the two-photon coincidence counting distribution for the two-photon entangled source. For comparison, we also use a laser beam with the same wavelength of 800nm to replace the down-converted beams and impinge on the circular aperture. The diffraction pattern is recorded by a CCD camera set at the back focal plane of the lens. The conventional Airy disk has been observed in Fig. 2(b). We can see that the two-photon Airy disk in Fig. 2(a) has a more subtle central spot with the half size of that for the classical Airy disk

in Fig. 2(b). For the both cases, the theoretical curves fit well with the experimental results.

In summary, we have experimentally observed the quantum Airy disk diffraction pattern with an entangled two-photon source. In contrast to the previous two-photon quantum lithography experiments where the far-field diffraction was observed, the present scheme is carried out by a convex lens to perform the Fraunhofer diffraction. The experimental result has demonstrated that the two-photon Airy disk can surpass the classical diffraction limit, achieving the super-resolution focusing effect. Therefore our experiment is not only interesting from a fundamental point of view, but also relevant for the quantum lithography application.

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